Problems from http://www.diophante.fr, due on that website by March 31, 2024

A2882

The tortoise challenges the hare in a race with two banners, the first 3 kilometers from the start and the second 5 kilometers from the start. The distances $L(t)$ and $T(t)$ covered in meters by the two competitors from the start obey the following formulas:

Hare	Tortoise
$L(t) = (t+1)^{\sqrt[3]{t}} - 1$	$T(t) = t^{\sqrt[3]{t+1}}$

where t is the number of hours after departure.

Q_1: Who starts the fastest?
Q_2: Who is in the lead at the first banner and at what time after the start to the nearest second?
Q_3: Who comes in first at the second banner and when, to the nearest second after the start?

A4957

As the prime number p takes on the values 2,3,5,7 and 11, Zig searches for all the prime numbers q and r not necessarily distinct such that the product qr divides $p^q + p^r$. Zig found 7 distinct ordered pairs (q,r) for a certain value of p. What is this value?

D1749

In an acute triangle ABC we trace the heights BD and CE.
The tangents at D and E to the circle circumscribed by triangle ADE meet at point P.
The bisector of AP meets the parallel to side BC passing through A at point Q
Demonstrate that the parallel passing through Q to the line [DE] divides the segments PD and PE in their middles.

D4936

We draw in the plane a square OABC with A on the positive x-axis and C on the positive y-axis such that OA = OC = k.

Q_1: Determine the smallest integer k such that we can divide this square into nine rectangles, at least one of which is strictly interior to the square and whose centers, all having integer coordinates, are the vertices of a convex 9-gon.
Q_2: Prove that we can find an integer k such that the square is cut into several rectangles whose side lengths are integers and the centers are the vertices of a convex23-gon.

E463

Diophante writes the integer $n > 0$ on the blackboard. Zig and Puce take turns multiplying the last integer written on the board by any integer of their choice ranging from 2 to 9. The first to reach a million or more wins the game. Determine the winner in the following three cases:
Q_1: $n = 1$. Zig starts the game.
Q_2: $n = 13$. Puce starts the game.
Q_3: $n = 2024$. Zig starts the game.

G2985

Let p be a natural whole number greater than 1. We consider a subset X of the set $E = \{1, 2, 3, \ldots, 2^p\}$, which contains the maximum number N_p of integers x such that if x belongs to X, then $2x$ does not belong to X.
Determine p in the following two cases:
Q_1: $N_p = 699051$
Q_2: $N_p = 22369621$